ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

HAMILTONIAN LACEABILITY IN MIDDLE GRAPHS

Manjunath.G*, Murali.R, B. Shanmukha
*Department of Mathematics,Gopalan college of Engineering and Management, Bangalore, India
2Department of Mathematics, Dr.Ambedkar Institute of Technology, Bangalore, India
3Department of Mathematics, PES College of Engineering, Mandya ,India

ABSTRACT

This paper is aimed to discuss Hamiltonian laceability in the context of the Middle graph of a graph. We explore laceability properties of the Middle graph of the Gear graph, Fan graph, Wheel graph, Path and Cycle.

KEYWORDS: Connected graph, Middle graph, Gear graph, Fan graph, Hamiltonian- $t*$ -laceable graph, Hamiltonian-t-laceability number $(\lambda_{(t)})$, $t*$-Connected graph.

INTRODUCTION

All graphs considered in this paper are finite, simple, connected and undirected. Let u and v be two vertices in a graph G. The distance between u and v denoted by $d(u,v)$ is the length of a shortest u - v path in G. G is a Hamiltonian- t -laceable if there exists in it a Hamiltonian path between every pair vertices u and v with the property $d(u,v)=t$,$1\le t\le diamG$, where t is a positive integer. If this property is achieved for at least one pair of vertices u and v, the graph G is termed Hamiltonian- $t*$ -laceable. Various results on Hamiltonian laceability properties in graphs are available in [4], [7], [8], [9], [10], [11], [12] and [13].

In this paper we give some results related to the Laceability properties of the Middle graph of the Gear graph (G_n), Fan graph $(F_{1,n})$, Wheel graph $(W_{1,n})$, Path graph (P_n) and Cycle (C_n).

The vertex set of G and the edge set of G are denoted respectively by $V(G)$ and $E(G)$ respectively. Terms not defined here can be found in [1].

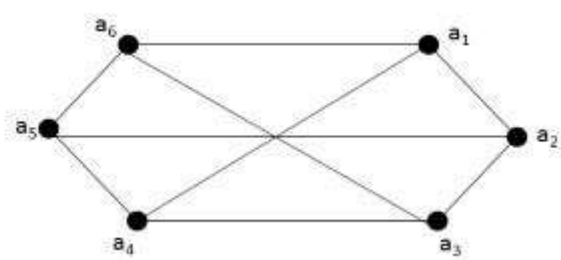

Figure 1: Hamiltonian Laceable graph

Definition 1

Let P be a path from the vertices a_i to a_j in a graph G and let P' be a path from a_i to a_k. Then the path $P\cup P'$ is the path obtained by extending the path P from a_i to a_j to a_i to a_k through the common vertex a_j (i.e., if $P: a_i......a_j$ and $P': a_j......a_k$, then $P\cup P': a_i...a_j....a_k$).

Figure 2 below illustrates a Hamiltonian-2-laceable graph and a Hamiltonian-2*-laceable graph.

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

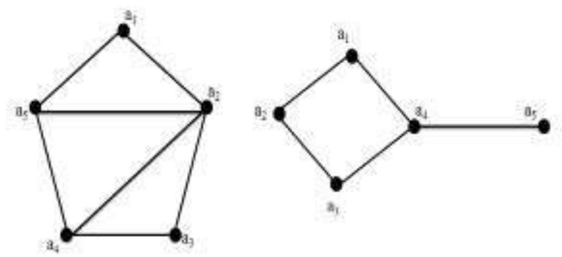

Figure 2: ***A Hamiltonian-2-laceable graph and a Hamiltonian-2*-laceable Graph***

Definition 2

Let a_i and a_j be any two distinct vertices in a connected graph G. Let E' be any minimal set of edges not in G and P be a path in G, such that $P \cup E'$ is a Hamiltonian Path in G from a_i and a_j. Then $|E'|$ is called the t-laceability number $\lambda_{(t)}$ of G and the edges in E' are called the t-laceability edges with respect to (a_i, a_j).

Definition 3

A graph G is $t*$-connected if it is Hamiltonian-$t*$-laceable for all $t, 1 \le t \le diamG.$

Definition 4

The Middle graph of G denoted by $M(G)$ is defined as follows. The vertex set of $M(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one of the following holds:

i) x, y are in $E(G)$ and x, y are in adjacent in G.

ii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

Figure 3 below illustrates the Middle graph of a cubic graph G.

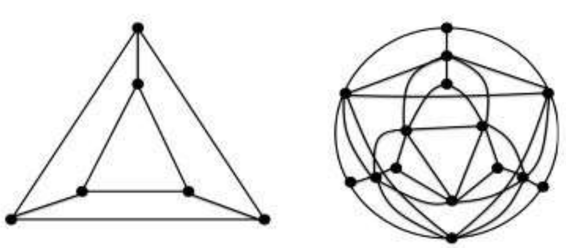

Figure 3: ***A Cubic Graph and its Middle Graph***

Definition 5

The Gear graph G_n is a wheel graph $W_{1,n}$ with a vertex added between each pair of adjacent vertices of the outer cycle.

Figure 4 below shows the graph G_6.

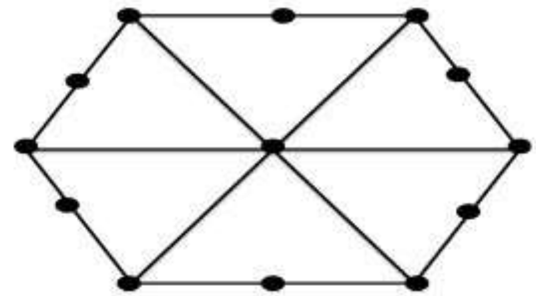

Figure 4: Gear Graph G_6

Definition 6

A Fan graph $F_{m,n}$ is defined as the graph join $\overline{K}_m + P_n$ where $\overline{K}_m$ is the empty graph on m vertices and P_n is the path graph on n vertices

The Fan graph $F_{1,n}$ is illustrated in Figure 5 below.

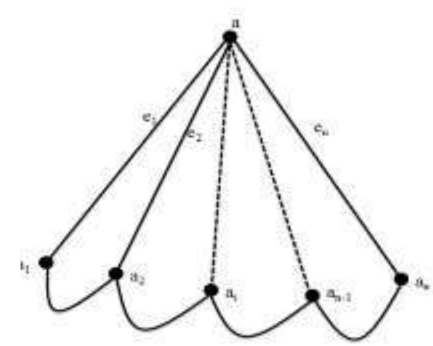

Figure 5: ***Fan Graph*** $F_{1,n}$

RESULTS

Theorem 1: The Graph $G = M(G_n)$, $n \ge 4$ is $t*$-connected with $\lambda_{(t)} = 1$

Proof: Let $V(G_n) = \{a\} \cup \{a_1, a_2, a_3, ..., a_{2n}\}$ and

$E(G_n) = \{e_i : 1 \le i \le n\} \cup \{e'_i : 1 \le i \le 2n-1\}$ $\cup \{e_n^1\}$ where e_i the edge is $a\, a_{2i-1} (1 \le i \le n)$, e'_i is the edge $a_i a_{i+1} (1 \le i \le 2n-1)$ and e_{2n}^1 is the

edge $v_{2n-1}v$, by the definition of Middle graph $V(M(G_n)) = V(G_n) \cup E(G_n) = \{a\} \cup \{a_i : 1 \le i \le 2_n\} \cup \{e_i : 1 \le i \le n\} \cup \{e'_i : 1 \le i \le 2_n\}$ respectively.
Clearly $d(G) = 4$.

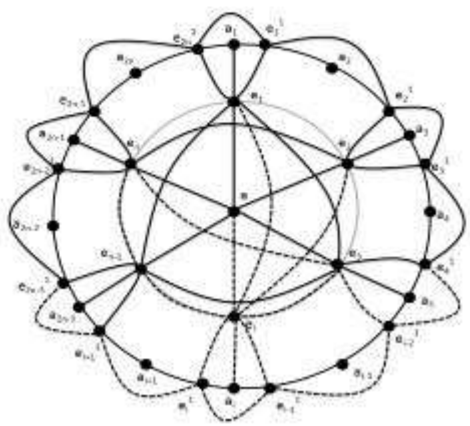

Figure 6: *Middle graph of Gear Graph* $M(G_n)$

Case (i): For $t = 1$

In the graph G, $d(a_1, e_1) = 1$ and the path P: $(a_1, e'_1) \cup (e'_1, e'_{2n-2}) \cup (e'_{2n-2}, a_{2n-2}) \cup (a_{2n-2}, e'_{2n-3}) \cup (e'_{2n-3}, a_{2n-3}) \cup (a_{2n-3}, e'_{2n-4}) \cup (e'_{2n-4}, a_{2n-4}) \cup$ ....$\cup$ $(e'_8, a_8) \cup (a_8, e'_7) \cup (e'_7, a_7) \cup (a_7, e'_6) \cup (e_6, a_6) \cup (a_6, e'_5) \cup (e'_5, a_5) \cup (a_5, e'_4) \cup (e'_4, a_4) \cup (a_4, e'_3) \cup (e'_3, a_3) \cup (a_3, e'_2) \cup (e'_2, a_2) \cup (a_2, e_2) \cup (e_2, e_3) \cup (e_3, e_4) \cup (e_4, e_5) \cup (e_5, e_6) \cup (e_6, e_7) \cup (e_7, e_8) \cup$................ ....$\cup (e_{n-1}, a) \cup (a, e_1)$. is a Hamiltonian path from a_1 to e_1. Where (a_2, e_2) is the laceability edge. Hence G is a Hamiltonian-1*-laceable.

Case (ii): For $t = 2$

In the graph G, $d(a_1, a_2) = 2$ and the path P: $(a_1, e'_1) \cup (e'_1, e_1) \cup (e_1, a) \cup (a, e_2) \cup (e_2, e_3) \cup (e_3, e_4) \cup (e_4, e_5) \cup (e_5, e_6) \cup (e_6, e_7) \cup$ .....$\cup (e_{n-1}, e_{n-2}) \cup (e_{n-2}, v_{2n-2}) \cup (e'_{2n-3}, a_{2n-3}) \cup (a_{2n-3}, e'_{2n-4}) \cup (e'_{2n-4}, a_{2n-4}) \cup$..........$\cup (e'_{n+1} a_{n+1}) \cup a_{n+1}, e'_n) \cup (e'_n, a_n) \cup (a_n, e'_8) \cup (e'_8, a_8) \cup (a_8, e'_7) \cup (e'_7, a_7) \cup (a_7, e'_6) \cup (e'_6, a_6) \cup (a_6, e'_5) \cup (e'_5, a_5) \cup (a_5, e'_4) \cup (e'_4, a_4) \cup (a_4, e'_3) \cup (e'_3, a_3) \cup (a_3, e'_2) \cup (e'_2, a_2)$. is a Hamiltonian path from a_1 to a_2. Where (e_{n-1}, e'_{2n-2}) is the laceability edge. Hence G is a Hamiltonian-2*-laceable.

Case (iii): For $t = 3$

In the graph G, $d(a_1, a_5) = 3$ and the path P: $(a_1, e'_1) \cup (e'_1, e'_{2n-2}) \cup (e'_{2n-2}, a_{2n-2}) \cup (a_{2n-2}, e'_{2n-3}) \cup (e'_{2n-3}, a_{2n-3}) \cup (a_{2n-3}, e'_{2n-4}) \cup (e'_{2n-4}, a_{2n-4}) \cup$ ....$\cup$ $(e'_{n+1} a_{n+1}) \cup (a_{n+1}, e'_n) \cup (e'_n, a_n) \cup$........ $\cup (e'_8, a_8) \cup (a_8, e'_7) \cup (e^1_7, a_7) \cup (a_7, e'_6) \cup (e'_6, a_6) \cup (a_6, e'_5) \cup (e'_5, e_3) \cup (e_3, a) \cup (a, e_4) \cup (e_4, e_{n-4}) \cup (e_{n-4}, e_{n-3}) \cup (e_{n-3}, e_{n-2}) \cup (e_{n-2}, e_{n-1}) \cup (e_1, a_2) \cup (a_2, e'_2) \cup (e'_2, e_2) \cup (e_2, a_3) \cup (a_3, e'_3) \cup (e'_3, a_4) \cup (a_4, e'_4) \cup (e'_4, a_5)$ is a Hamiltonian path from a_1 to a_5. Where (b_1, a_2) is the laceability edge. Hence G is a Hamiltonian-3*-laceable.

Case (iv): For $t = 4$

In the graph G, $d(a_1, a_4) = 4$ and the path P: $(a_1, e'_1) \cup (e'_1, a_2) \cup (a_2, e'_2) \cup (e'_2, a_3) \cup (a_3, e'_3) \cup (e'_3, e'_4) \cup (e'_4, a_5) \cup (a_5, e'_5) \cup (e'_5, a_6) \cup (a_6, e'_6) \cup (e'_6, a_7) \cup (a_7, e'_7) \cup (e'_7, a_8) \cup (a_8, e'_8) \cup$ ....$\cup (a_{n+2}, e'_{n+2}) \cup (e'_{n+2}, a_{n+3}) \cup (a_{n+3}, e'_{n+3}) \cup$..............$\cup (a_{2n-2}, e'_{2n-2}) \cup (e'_{2n-2}, e_1) \cup (e_1, a) \cup (a, e_{n-1}) \cup (e_{n-1}, e_{n-2}) \cup (e_{n-2}, e_{n-3}) \cup (e_{n-3}, e_{n-4}) \cup (e_{n-4}, e_{n-5}) \cup$................ $(e_4, e_3) \cup (e_3, e_2) \cup (e_2, a_4)$ is Hamiltonian path from a_1 to a_4. Where (e_2, a_4) is the laceability edge. Hence G is a Hamiltonian-4*-laceable. Hence the proof.

Theorem 2. The Graph $G = M(F_{1,n})$, $n \geq 3$ is t^* -connected.

Proof: Consider the Fan graph $F_{1,n}$ denote the vertices $V(F_{1,n})$ = $\{a\} \cup \{a_1, a_2, a_3, ..., a_{n-1}, a_n\} \cup \{b_1, b_2, b_3 ...., b_{n-1}, b_n\}$ and $E(F_{1,n}) = \{e_i : 1 \leq i \leq n\}$

Where e_i is the edge $aa_i (1 \leq i \leq n)$ by the definition of middle graph $V(M(F_{1,n})) = V(F_{1,n}) \cup E(F_{1,n})$ = $\{a_i : 1 \leq i \leq n\} \cup \{b_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\}$. Clearly $d(G) = 2$.

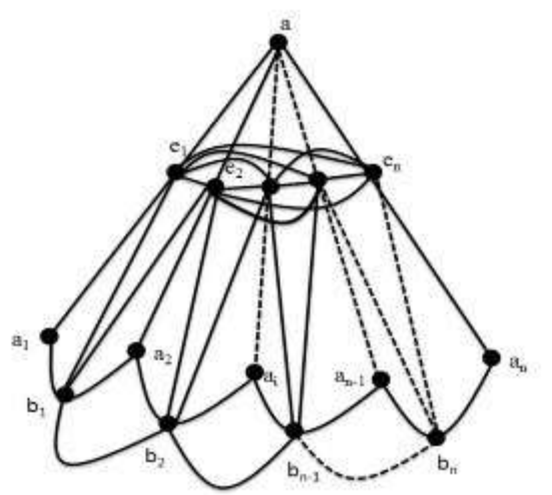

Figure 7: ***Middle Graph of Fan Graph*** $M(F_{1,n})$

Case (i): For $t = 1$

In the Graph G, $d(a, e_1') = 1$ and the path $P: (a, e_2) \cup (e_2, e_3) \cup (e_3, e_4) \cup (e_4, e_5) \cup (e_5, e_6) \cup .... \cup (e_{n-1}, e_n) \cup (e_n, a_n) \cup (a_n, u_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup (b_{n-3}, a_{n-3}) \cup .... \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, a_2) \cup (a_2, b_1) \cup (b_1, a_1) \cup (a_1, e_1)$ is Hamiltonian path from a to e_1'.

Hence G is a Hamiltonian-1-laceable.

Case (ii): For $t = 2$

In the Graph G, $d(a, a_1) = 2$ and the path $P: (a, e_2) \cup (e_2, e_3) \cup (e_3, e_4) \cup (e_4, e_5) \cup (e_5, e_6) \cup .... \cup (e_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup (b_{n-3}, a_{n-3}) \cup .... \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, a_2) \cup (a_2, b_1) \cup (b_1, a_1)$ is Hamiltonian path from a to a_1. Hence G is a Hamiltonian-2-laceable.

Hence the proof

Theorem 3. The Graph $G = M(w_{1,n})$, $n \geq 3$ is t^* -connected.

Proof: Consider Wheel graph $W_{1,n}$ denote the vertices $\{a\} \cup \{a_1, a_2, a_3, ..., a_n\} \cup \{b_1, b_2, b_3 ...., b_n\}$ $V(W_{1,n})$ =and $E(W_{1,n}) = \{e_i : 1 \leq i \leq n\} \cup \{e_n\}$

Where e_i is the edge $a\, a_{i+1} (1 \leq i \leq n-1)$, e_n is the edge $a_n a_1$, by the definition of middle graph $V(M(W_{1,n})) = V(W_{1,n}) \cup E(W_{1,n})$ = $\{a_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\}$ .Clearly $d(G) = 3$.

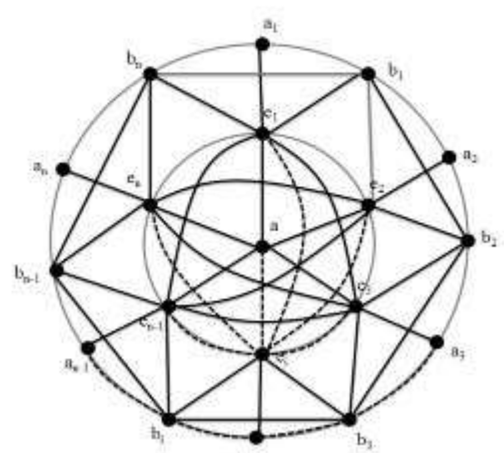

Figure 8: Middle Graph of Wheel Graph $M(W_{1,n})$

Case (i): For $t = 1$

In the Graph G, $d(a_1, b_1) = 1$ and the path $P: \{(a_1, e_1) \cup (e_1, a) \cup (a, e_3) \cup (e_3, e_4) \cup (e_4, e_5) \cup (e_5, e_6) \cup .... \cup (e_n, b_n) \cup$

$(b_n,a_n)\cup$ $(a_n,b_{n-1})\cup$ $(b_{n-1},a_{n-1})\cup$ $(a_{n-1},b_{n-2})\cup$ $(b_{n-2},a_{n-2})\cup$ $(a_{n-2},b_{n-3})\cup$ $(b_{n-3},a_{n-3})\cup.......\cup$ $(b_4,a_4)\cup$ $(a_4,b_3)\cup$ $(b_4,a_3)\cup$ $(a_3,b_2)\cup$ $(b_2,a_2)\cup$ $(a_2,e_2)\cup$ (e_2,b_1) is Hamiltonian path from a_1 to b_1. Hence G is a Hamiltonian-1*-laceable.

Case (ii): For $t=2$
In the Graph G, $d(a_1,b_2)=2$ and the path and the path $P:(a_1,b_1)\cup(b_1a_2)\cup(a_2,e_2)\cup(e_2,a)\cup$ $(a,e_1)\cup$ $(e_1,b_n)\cup$ $(b_n,e_n)\cup$ $(e_n,a_n)\cup$ $(a_n,b_{n-1})\cup$ $(b_{n-1},e_{n-1})\cup$ $(e_{n-1},a_{n-1})\cup$ $(a_{n-1},b_{n-2})\cup$ $(b_{n-2},e_{n-2})\cup$ $(e_{n-2},a_{n-2})\cup$ $(e_{n-2},b_{n-3})\cup$ $(b_{n-3},e_{n-3})\cup$ $(e_{n-3},a_{n-3})\cup......$ $(b_4,e_4)\cup$ $(e_4,a_4)\cup$ $(a_4,b_3)\cup$ $(b_3,a_3)\cup$ $(a_3,e_3)\cup(e_3,e_2)$ is Hamiltonian path from a_1 to b_2. Hence G is a Hamiltonian-2*-laceable.

Case (iii): For $t=3$
In the Graph G, $d(a_1,a_3)=3$ and the path $P:$ $(a_1,e_1)\cup(e_1,a)\cup(a,e_4)\cup(e_4,e_5)\cup(e_5,e_6)$ $\cup............\cup(e_n,b_n)\cup$ $(b_n,a_n)\cup$ $(a_n,b_{n-1})\cup$ $(b_{n-1},a_{n-1})\cup$ $(a_{n-1},b_{n-2})\cup$ $(b_{n-2},a_{n-2})\cup$ $(a_{n-2},b_{n-3})\cup$ $(b_{n-3},a_{n-3})\cup.........\cup$ $(b_4,a_4)\cup$ $(a_4,b_3)\cup$ $(b_3,e_3)\cup$ $(e_3,e_2)\cup$ $(e_2,b_1)\cup$ $(a_2,e_2)\cup$ $(e_2,b_1)\cup$ $(b_1,v_2)\cup$ (a_2,b_2) $\cup(b_2,a_3)$ is Hamiltonian path from a_1 to b_3. Hence G is a Hamiltonian-3*-laceable.

Theorem 4: The Graph $G=M(P_n)$, $n\geq 3$ is Hamiltonian- t^* -laceable for $t=1,2,3$ with $\lambda_{(t)}=1$.

Proof: Consider the graph G can be obtained by the Middle graph $M(P_n)$. Since $M(P_n)$ contains at least one cycle of length 3, we can conclude that $G=M(P_n)\geq 3$.

Let $V(P_n)=\{a_1,a_2,a_3,.....,a_n\}$ and $V(M(P_n))=\{a_i:1\leq i\leq n\}\cup\{b_i:1\leq i\leq n-1\}$ where b_i is the vertex of $M(P_n)$ corresponding to edge a_ia_{i+1} of P_n. Clearly $d(G)=n$.

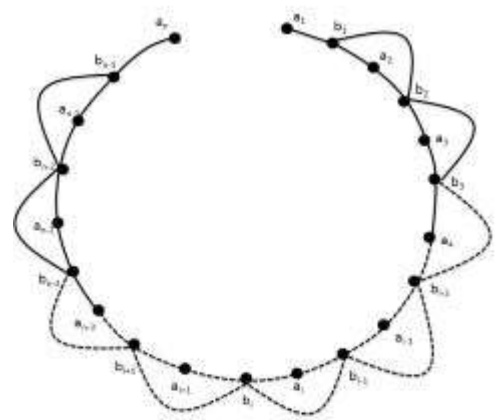

Figure 9: ***Middle Graph of Path*** $M(P_n)$

Case (iii): For $t=1$
In the graph G, $d(a_1,b_1)=1$, and the path $P:$ $(a_1,a_n)\cup$ $(a_n,b_{n-1})\cup$ $(b_{n-1},a_{n-1})\cup$ $(a_{n-1},b_{n-2})\cup$ $(b_{n-2},b_{n-3})\cup$ $(b_{n-3},a_{n-3})\cup$ $..........\cup(b_3,a_3)\cup$ $(b_2,a_2)\cup$ (a_2,b_1) is Hamiltonian path from a_1 to b_1. Where (a_1,a_n) is the laceability edge. Hence G is a Hamiltonian-1*-laceable.

Case (ii): For $t=2$
In the graph G, $d(a_1,a_2)=2$, and the path $P:$ $(a_1,b_1)\cup$ $(b_1,b_2)\cup$ $(b_2,a_3)\cup$ $(a_3,b_3)\cup$ $(b_3,a_4)\cup$ $(a_4,b_4)\cup...................\cup$ $(u_{n-3},v_{n-2})\cup$ $(b_{n-2},a_{n-1})\cup$ $(a_{n-1},b_{n-1})\cup$ $(b_{n-1},a_n)\cup(a_n,a_2)$ is Hamiltonian path from a_1 to a_2. Where (a_n,a_2) is the laceability edge. Hence G is a Hamiltonian-2*-laceable.

Case (iii): For $t=3$
In the graph G, $d(a_1,a_3)=3$, and the path $P:$ $(a_1,b_1)\cup$ $(b_1,a_2)\cup$ $(a_2,b_2)\cup$ $(b_2,b_3)\cup$ $(b_3,a_4)\cup$ $.......................\cup(b_{n-3},a_{n-2})\cup$ $(a_{n-2},b_{n-2})\cup$ $(b_{n-2},a_{n-1})\cup$ $(a_{n-1},b_{n-1})\cup$ $(b_{n-1},a_n)\cup(a_n,a_3)$

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

is Hamiltonian path from a_1 to a_3. Where (a_n, a_3) is the laceability edge. Hence G is a Hamiltonian-3*-laceable. Hence the proof.

Theorem 5: The Graph $G = M(C_n)$, $n \geq 3$ is
(i) Hamiltonian-t*-laceable for $t = 1$ and 2
(ii) Hamiltonian- t* -laceable for $t = 3$, with $\lambda_{(t)} = 1$

Proof: $V(C_n) = \{a_1, a_2, a_3, ..., a_n\}$ and $V(M(C_n)) = \{a_1, a_2, a_3, ..., a_n\}$ $\cup$ $\{b_1, b_2, b_3, ..., b_n\}$ where, b_i is the vertex of corresponding to the edges $a_i a_{i+1}$ of $C_n (1 \leq i \leq n-1)$.

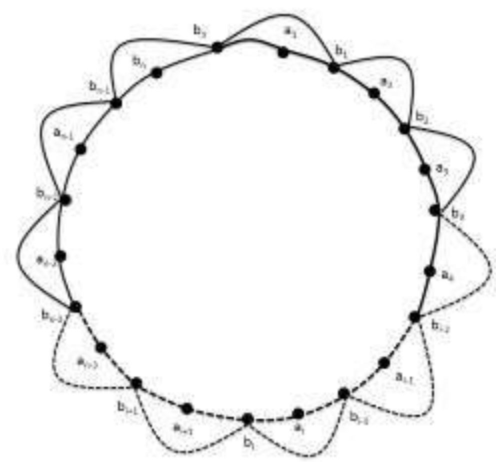

Figure 10: ***Middle Graph of*** $M(C_n)$

Case (iii): For $t = 1$
In the graph G, $d(a_1, b_1) = 1$ and the path P: $(a_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup .... \cup (b_{i+1}, a_{i+1}) \cup (b_i, a_i) \cup (a_i, b_{i-1}) \cup (b_{i-1}, a_{i-1}) \cup .... \cup (b_5, a_5) \cup (a_5, b_4) \cup (b_4, a_4) \cup (a_4, b_3) \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, a_2) \cup (a_2, b_1)$ is Hamiltonian path from a_1 to b_1. Hence G is a Hamiltonian-1*-laceable.

Case (ii): For $t = 2$
In the graph G, $d(a_1, a_2) = 2$ and the path P: $(a_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup .... \cup (b_{i+1}, a_{i+1}) \cup (b_i, a_i) \cup (a_i, b_{i-1}) \cup (b_{i-1}, a_{i-1}) \cup ....$ $.................... \cup (b_5, a_5) \cup (a_5, b_4) \cup (b_4, a_4) \cup (a_4, b_3) \cup (b_3, a_3) \cup (a_3, b_2) \cup (b_2, b_1) \cup (b_1, a_2)$ is Hamiltonian path from a_1 to a_2. Hence G is a Hamiltonian-2*-laceable.

Case (iii): For $t = 3$
In the graph G, $d(a_1, a_3) = 3$ and the path P: $(a_1, b_n) \cup (b_n, a_n) \cup (a_n, b_{n-1}) \cup (b_{n-1}, a_{n-1}) \cup (a_{n-1}, b_{n-2}) \cup (b_{n-2}, a_{n-2}) \cup (a_{n-2}, b_{n-3}) \cup ..... \cup (b_{i+1}, a_{i+1}) \cup (b_i, a_i) \cup (a_i, b_{i-1}) \cup (b_{i-1}, a_{i-1}) \cup .... \cup (b_5, a_5) \cup (a_5, b_4) \cup (b_4, a_4) \cup (a_4, b_3) \cup (b_3, b_2) \cup (b_2, a_2) \cup (v_2, u_1) \cup (b_1, b_3)$ is Hamiltonian path from a_1 to a_3. Where (b_1, a_3) is the laceability edge. Hence G is a Hamiltonian-3*-laceable.

ACKNOWLEDGEMENTS

The first author is thankful to the Management and Staff of the Department of Mathematics, Gopalan College of Engineering and Management, Bangalore for their support and encouragement. The authors are also thankful to the Management and R & D Centre, Department of Mathematics, Dr. Ambedkar Institute of Technology, Bangalore and the Department of Mathematics, PES College of Engineerring, Mandya.

REFERENCES

1. F. Harary, Graph Theory, Narosa Publishing 1969.
2. D. Michalak, On Middle and Total graphs with Coarseness Number equal 1, Lecture note s in Mathematics, Volume 1018, Graph theory, Springer-Verlag, Berlin,139-150(1983).
3. Jain Akiyama and Takashi Hamada and I.Yoshimura, On

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

Charecterization of middle g raphs, True Mathematics 11 (1975),pp 35-39.

4. Thimmaraju S. N and Murali. R, Laceability On a Class of Regular Graphs, Internationa Journal of Comput ational Science and Mathemati cs,2010:2(3):397-406.
5. Vernold Vivin. J and K. Thilagavathi, On Harmonious Colouring of Line Graph of Central Graph of Paths, Applid MathematicalmSci ences Vol.3,2009, No.5, 205-214.
6. S. K. Vaidya and P. L. Vihol, Cardial Labeling for Middle Graph of Some Graphs,Discrete Mathematics E lixir Dis.Math.34 C(2011) 2468-2476.
7. Girisha. A and R. Murali, i-Hamiltonian Laceability in Some Product Graphs, Internationa Journal of Computational Science and Mahematics, ISSN 0974- 3189, Volume 4 No. 2(2012),pp 145- 158.
8. Girisha. A and R. Murali, Hamiltonian Laceability in Some Classes of Star Graphs ISSN 2319-5967 IJESIT Volume 2, Issue 3, May 2013.
9. Manjunath.G and R. Murali, Hamiltonia n Laceability in Brick Product C(2n+1,1, r),Advances in Applied Mathematical Bioscie nces, ISSN 2248-9983, Volume5, No.1(2014), pp.13-32.
10. Manjunath. G, R. Murali and S. N. Thimmaraju, Hamiltonian Laceability in Modified Brick Product of Odd Cycles, IJMS, Accepted.
11. Manjunath. G and R. Murali, Hamiltonian-t*-laceability in Jump Graphs of Diameter two, IOSR Journal of Mathematics,e-2278-3008,p- ISSN:2319-7676.Volume10,Issue 3Ver.III(May Jun.20 14), pp-55-63.
12. Manjunath.G,R.Murali and Girish .A, Hamiltonian Laceability in Line Graphs, International Journal of Computer Application (IJCA).Volume 98, Number 12, pp.17-25.
13. Manjunath. G, R. Murali and Rajendra. S. K, Hamiltonian Laceability in Total Graphs, International Journal of Mathem atics and Research, Volume 2, Issue 12 (Dec 2014), pp.774-785 ISSN:2320- 7167.